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ON ELASTIC AND INELASTIC HEAVY ION SCATTERING IN THE HIGH-ENERGY APPROXIMATION

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Using the high-energy approximation method for the three-dimensional quasi-classics, elastic and inelastic cross sections of heavy ions at large angles are calculated. The role of the deflection angle introduced in the theory and of the parameters of an interaction is discussed. The corresponding amplitudes are obtained in analytic forms and a good agreement with experimental data is also obtained.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Упругое и неупругое рассеяние тяжелых ионов в высокоэнергетическом приближении

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Дифференциальные сечения упругого и неупругого рассеяния тяжелых ионов на ядрах рассчитаны на основе высокоэнергетического приближения для квазиклассического рассеяния в поле комплексного потенциала. Проанализирована роль отклонения траектории от прямой линии. Амплитуды рассеяния получены в аналитическом виде. Достигнуто хорошее согласие результатов расчета с экспериментальными данными.

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1. Introduction

Elastic and inelastic scattering of alpha-particles, light and heavy ions on nuclei at energies $E >> V$ is very sensitive to the parameters of an interaction potential and also to the detailed behavior of the structure characteristic such as the density distributions, transition matrix elements and so on. Indeed, in this case the corresponding wave length λ is much smaller than typical dimensions of a nucleus, the radius R and thickness a of a boundary of the nuclear interaction. Moreover, a specific problem appears at large scattering angles when the cross section is as a rule exponentially decreasing because the sets of partial wave decompositions become the sign-

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alternative ones and, therefore, one needs to keep in the computer memory a lot of partial phases. One of the ways to decrease these difficulties is to use one-dimensional quasi-classics for calculating the partial phases and then to develop special methods of summing up the corresponding partial sets [1]. However, the initial conditions $E >> V$, $kR >> 1$ may be used themselves for developing the approach where it is not necessary to use the partial wave expansion for the elastic and inelastic scattering amplitude. In particular, the method was developed based on three-dimensional quasi-classics that operates not with the one-dimensional partial waves but directly with the thereee-dimensional action function [2—4]. A deflection of the classical trajectory of motion on the straight line is included which plays an important role especially in the case of the heavy-ion scattering. We have used the realistic complex nuclear and Coulomb potentials and made comparison with experimental data obtained for the heavy-ion beams at energies about one hundred MeV per nucleon. Below we apply this method to the processes of elastic and inelastic scattering

2. Elastic Scattering

Heavy ion elastic scattering at energies larger than several dozen MeV per nucleon is just the process to which the method mentioned above can be adjusted. To this aim we use the elastic scattering amplitude obtained in [3] for large angles $\theta > (1/kR)$ and $\theta > \theta_c \cong (|V|/E)$ covering in practice a wide region of scattering angles

$$T^{el} = -\frac{m}{2\pi h^2}\int d\mathbf{r}\Psi_{\mathbf{k}_f}^{(-)*} V \Psi_{\mathbf{k}_i}^{(+)} =$$

$$= -\frac{m}{2\pi h^2}\int d\mathbf{r}(V_N + V_C)\exp\{i\,\mathbf{q}_{ef}\mathbf{r} + i\,\Phi(\mathbf{r})\}, \qquad (2.1)$$

where [2]

$$\Phi = \Phi_i^{(+)} + \Phi_f^{(-)};$$

$$\Phi^{(\pm)} = -\frac{1}{hv}\int_{\mp z}^{\infty}[V_N(\sqrt{\rho^2+\lambda^2}) + V_C(\sqrt{\rho^2+\lambda^2})]d\lambda, \qquad (2.2)$$

and the potentials

$$V_N = V + iW = V_0 f_V(r) + iW_0 f_W(r), \qquad (2.3)$$

$$V_C = \frac{Z_1 Z_2 e^2}{R_C} \int \frac{\rho_c(x)dx}{|\mathbf{r} - \mathbf{x}|}, \quad \rho_c = \rho_0 f_c(r) \tag{2.4}$$

with the charge density distribution $\rho_c(r)$ and the effective momentum transfer $\mathbf{q}_{ef} = \mathbf{q} - \mathbf{q}_c$, where $\mathbf{q} \| \mathbf{q}_c$, $q_{ef} = 2k(\alpha - \alpha_c)$, $\alpha = \sin(\theta/2)$, and $\alpha_c \cong \frac{1}{2E}\,[V(R_t) + V_c(R_t) + iW(R_t)]$, taken at the radius R_t of the external limited trajectory of motion. All the distribution functions are taken in the form of the Fermi-function

$$f_p(r) = \frac{1}{1 + \exp\dfrac{r - R_p}{a_p}}. \tag{2.5}$$

Thus, the scattering amplitude consists of three terms:

$$T^{el} = T_V^{el} + iT_W^{el} + T_C^{el}. \tag{2.6}$$

Substituting (2.4) into T_C^{el} we obtain the 6-dimensional integral. It can be transformed to the 3-dimensional one if one expands the phase Φ in $\mathbf{u} = \mathbf{r} - \mathbf{x}$ and then integrates over $d\mathbf{u}$ [5]

$$T_C^{el} = -\frac{m}{2\pi h^2} \int d\,\mathbf{r} v_c(r) \exp\{i\tilde{\Phi}(\mathbf{r})\}, \quad v_c(r) = \frac{Z_1 Z_2 e^2 \rho_0}{q_e^2 R_C} f_c(r),$$

$$\tilde{\Phi}(\mathbf{r}) = \mathbf{q}_{ef}\mathbf{r} + \Phi(\mathbf{r}), \tag{2.7}$$

where $q_e \cong q_{ef}$, and $v_c(r)$ plays the role of a quasi-potential of scattering on a spread nuclear charge. Now each of the terms of the scattering amplitude (2.6) has the same form:

$$T_P^{el} = -\frac{m}{2\pi h^2} \int d\,\mathbf{r} Y_p f_p(r) \exp\{i\tilde{\Phi}(\mathbf{r})\}, \tag{2.8}$$

where Y_p is the «strength» of the corresponding part of the whole potential.

It has been shown in [2,4] that in calculating the whole quasi-classical phase $\tilde{\Phi}$ one may limit oneself to the step nuclear potential and the inside-of-R part of a Coulomb potential. In this case the phase has a rather simple form with a typical power dependence on the variables of integration r, $\mu = \cos\bar{\theta}$ and $\cos\bar{\phi}$. It is written as follows:

$$\widetilde{\Phi} = 2\widetilde{a}_0 + \widetilde{\beta}\mu + n_1\mu^2 + c_1\mu^3 +$$

$$+ n_2(1-\mu^2)\cos^2\overline{\varphi} + c_2\mu(1-\mu^2)\cos^2\overline{\varphi}, \tag{2.9}$$

where $\widetilde{\beta}$, c and n are expressed through a_n. For example,

$$\widetilde{\beta} = 2k(\alpha - \alpha_c)r + \overline{\beta};$$

$$\overline{\beta} = -\frac{2}{h\nu}[(V_0 + iW_0) + \frac{Z_1 Z_2 e^2}{2R_C}(3 - \frac{r^2}{R_C^2})]\alpha\, r. \tag{2.10}$$

Now, keeping in mind that $dr = -r^2 dr d\mu d\overline{\varphi}$, one can integrate in (2.8) over $d\mu$ by parts

$$I = \int_{-1}^{+1} d\mu \exp[i\widetilde{\Phi}(r, \mu, \overline{\Phi})] = -i\frac{\exp(i\widetilde{\Phi})}{\partial\Phi/\partial\mu}\Bigg|_{-1}^{+1} +$$

$$+ i\int d\mu \exp(i\widetilde{\Phi})\frac{\partial^2\widetilde{\Phi}/\partial\mu^2}{(\partial\widetilde{\Phi}/\partial\mu)^2}, \tag{2.11}$$

neglecting the second term, having the smallness $(kR)^{-2}$. The result is

$$I = -i\exp(2i\overline{a}_0 + in_1)[I^{(+)} - I^{(-)}];$$

$$I^{(\pm)} = \frac{\exp[\pm i(\widetilde{\beta} + c_1)]}{\Delta_{(\pm)} \mp \delta_{(\pm)}\cos^2\overline{\varphi}}; \tag{2.12}$$

$$\Delta_{(\pm)} = \widetilde{\beta} + 3c_1 \pm 2n_1; \quad \delta_{(\pm)} = 2(n_2 \pm c_2). \tag{2.13}$$

Then the integration over $d\varphi$ is performed with the help of a table integral. Thus, we can write the amlitude (2.8) in the form of a one-dimensional integral [2]:

$$T_p^{el} = \frac{im}{\hbar^2} Y_p \int_0^\infty f_p(r)\left\{F_p^{(+)}(r) - F_p^{(-)}(r)\right\}dr, \tag{2.14}$$

where

$$F_p^{(\pm)}(r) = \frac{r\exp[\pm i(\tilde{q}r + c_1)]\exp[i(2\bar{a}_0 + n_1]}{\tilde{q}L(\pm)};$$

$$L(\pm) = \frac{1}{\beta}\sqrt{\Delta_{(\pm)}(\Delta_{(\pm)} - \delta_{(\pm)})}. \qquad (2.15)$$

Integration in (2.14) can be done [2,4] if one uses the properties of the Fermi-function, which has poles $r_n^{(\pm)} = R \pm i\pi a(2n+1)$, $(n = 0,1,2..)$ on the complex r-plane. In practice, for the typical nuclear parameters it is enough to take into account only a couple of poles $r_0^{(\pm)} = R \pm ix_0$ nearest to the real axis («two-pole approximation»), because every next pair contributes approximately an order smaller than the previous one. Then, we have

$$T_p^{el} = -\frac{im}{h^2} Y_p 2\pi ia[F^{(+)}(r_0^{(+)}) + F^{(-)}(r_0^{(-)})]. \qquad (2.16)$$

Substituting into (2.16) the corresponding poles one can easily find that the amplitude, roughly speaking, behaves as an exponential function, depending on the exponent $-2\pi ak\sin\theta/2$ and oscillating with a frequency as a function of the radius R.

3. Inelastic Scattering

For calculating the inelastic scattering of light and heavy ions with excitation of the collective nuclear states we have used DWBA with the relative-motion *QC*-wave functions whose phases are calculated as it is shown in Sec.2. The energy change in the *out*-channel is neglected since usually $E_{ex} \ll E$. The transition interaction is constructed as usual with the help of derivatives in small quadrupole and octupole additions $\delta R = R\sum \alpha_{LM} Y_{LM}^*(\hat{r})$ to the radius of a potential in the elastic channel. The result for the amplitude is the same as if one uses the sudden approximation

$$T^{in} = (J_f M_f | \hat{T}_V^{el} + i\hat{T}_W^{el} + \hat{T}_C^{el} | J_i M_i), \qquad (3.1)$$

where

$$\hat{T}_p^{el} = -\frac{m}{2\pi h^2}\int d\mathbf{r} Y_p f_p(r, R + \delta R)\Psi^{(-)*}\Psi^{(+)} \qquad (3.2)$$

is the operator, depending on the internal nuclear coordinates α_{LM}. Then, substituting (3.2) into (3.1) we get

$$T_p^{in} = \sum_{LM} (J_f M_f | \alpha_{LM} | J_i M_i) \tilde{T}^{in}_{(p)LM}, \tag{3.3}$$

where

$$\tilde{T}^{in}_{(p)LM} = - \frac{m}{2\pi h^2} Y_p R \int dr\, \Psi^{(-)*} \Psi^{(+)} d \frac{f_p}{dR} Y^*_{LM}. \tag{3.4}$$

Transforming the structure matrix element in (3.3) through the reduced one and using the definition of $B\downarrow(EL)$-transition, one can write the inelastic cross section:

$$\frac{d\sigma}{d\Omega} = \frac{(2J_f + 1)}{(2J_i + 1)} \frac{1}{(2L + 1)} \sum_{LM} \frac{B\downarrow(EL)}{D_L^2} |\tilde{T}^{in}_{LM}|^2 \tag{3.5}$$

with

$$D_L = Z_2 e \rho_0 R_C J_L^c; \quad J_L^c = \int \frac{df_c}{dR_C} r^{L+2} dr \cong R_C^{L+2}. \tag{3.6}$$

One can show that all the terms with $M \neq 0$ may be neglected because of the additional fast oscillations in integrands as compared with the term $M = 0$. Then, the principal difference of the inelastic amplitude from the elastic one appears in integral over $d\mu$, because now in the upper and lower limits $\mu = \pm 1$ we have to take into account the relation

$$Y_{L0}(-\mu) = (-1)^L Y_{L0}(+\mu), \tag{3.7}$$

which changes the sign of the second term in the inelastic analog of eq. (2.14) for odd L. Indeed, using the relation $df_p/dR = -df_p/dr$, we get:

$$\tilde{T}_p^{in} = -\frac{im}{h^2} Y_p Y_{L0}(1) R \int_0^\infty dr \frac{df_p}{dr} \left\{ F_p^{(+)}(r) - (-1)^L F_p^{(-)}(r) \right\}. \tag{3.8}$$

This integral can be calculated in an analytical form if one uses the second order poles on the complex plane of the derivative df/dr. However, we show another way. Indeed, bearing in mind that the $F^{(\pm)}$-functions rapidly oscillate with increasing r because of the exponent $\tilde{q}r \gg 1$, one can integrate in (3.8) by parts

$$\int_0^\infty \frac{df_p}{dr} F_p^{(\pm)} dr = -\int_0^\infty f_p \frac{dF_p^{(\pm)}}{dr} dr - O\left(\frac{1}{(\tilde{q}r)^2}\right). \tag{3.9}$$

So, substituting (3.9) into (3.8) we get a form like (2.14) for elastic scattering with some additions in the integrand, namely, the factor $\tilde{q}R$ and the multiplier $(-1)^L$ before the second term:

$$\tilde{T}_p^{in} = \frac{im}{h^2} Y_p Y_{L0}(1) R\tilde{q} \int_0^\infty dr f_p(r) \left(F_p^{(+)}(r) - (-)^L F_p^{(-)}(r)\right). \tag{3.10}$$

Subsequent calculations are the same as in the case of elastic scattering, using the two-pole approximation.

4. Conclusion

Calculations of differential cross sections for elastic and inelastic scattering within the two-pole approximation are presented in Figs.1 and 2 in comparison with the experimental data from [6]. One can see a rather good agreement in the range of scattering angles $\theta > \theta_c \cong 2°$ in coincidence with the initial assumptions of the HEA-method. For each set of colliding nuclei we got the same interaction parameters for elastic and inelastic channels excluding the absorption W_0 that occurred to be about 10% as small as that in the elastic channel. The depths of potential wells are in

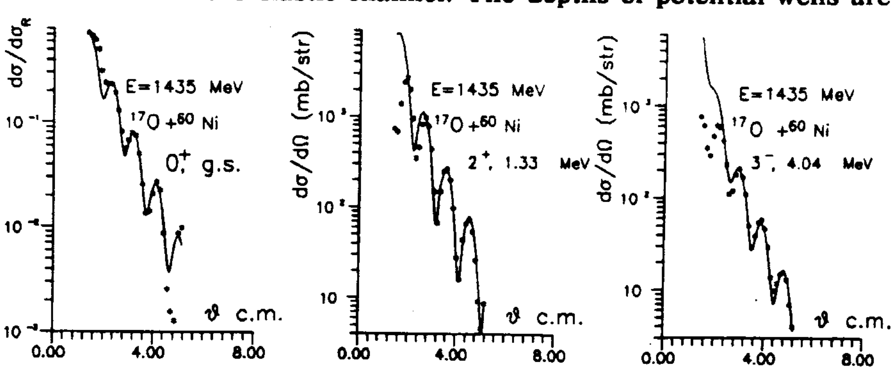

Fig.1. The heavy ion elastic and inelastic cross sections $^{17}O+^{60}Ni$; E_{lab} = 1435 MeV; exp. data from [6]; solid lines — theory

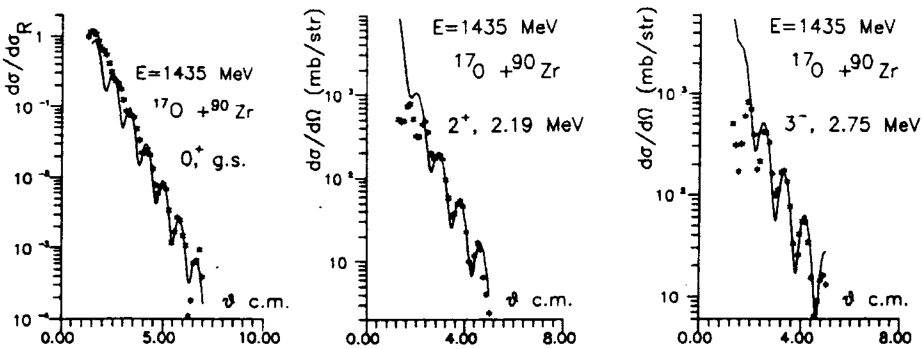

Fig.2. The same as in Fig.1, but for $^{17}O+^{90}Zr$

limits of V_0 = 60—70 MeV and W_0 = 5—6 MeV, the $B(EL)$-transitions obtained are approximately twice those cited in [6]. The most interesting result is that the thickness parameters for inelastic channels are about two-three times as small as those for elastic channels, where we have a_{el} = 0.55—0.6 fm. This might signify that in collective excitations of nuclei not all the particle states take part in forming the transition matrix elements. Otherwise, in elastic scattering the «tail» of a potential is formed from the whole set of one-particle states. It is easy to see from (3.10) that the oscillating part of the amplitude as a function of the scattering angle is the cos- or sin-function depending an L-even or odd, respectively. In this case the cross section will have visible oscillations which coincide for excitations of the even collective states in their phases with the elastic scattering oscillations. We can summarize that investigations of heavy ion collisions in the quantum region of scattering angles $\theta > \theta_c$, outside the limited trajectories of motion, are very sensitive to the precise structure of a nuclear-nuclear interaction. For instance, the slope of curves with θ feels the «thickness» of the acting region in the corresponding channel. This may be used also in searching for the «halo» distributions of nuclei in the radioactive beams which now become available. We hope that the HEA-method suggested can be successfully used in both the qualitative and quantitative analyses of scattering processes and direct reactions.

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Reference

1. Ford K.W., Wheeler J.A. — Ann. Phys., 1959, 7, p.259; Zagrebaev V.I. — Ann. Phys., 1990, 197, p.33.
2. Lukyanov V.K. — JINR E4-94-314, Dubna, 1994.

3. Lukyanov V.K. — Bull. of Russ. Acad. of Sc., Physics, 1994, 58, p.6.
4. Lukyanov V.K. — Proc. Int. School-Seminar on HI Phys., Dubna, 1993, v.2, p.129; JINR E7-93-274, Dubna, 1993; Lukyanov V.K., Gridnev K.A., Embulaev A.V. — Bull. of Russ. Acad. of Sc., Physics, 1994, 58, p.19.
5. Lukyanov V.K., Pol Yu.S. — Sov. J. Particles Nucl., 1975, 5, p.385.
6. Liguori Neto R. et al. — Nucl. Phys., 1993, A.560, p.733.

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